The efficient settlement of many-way debts

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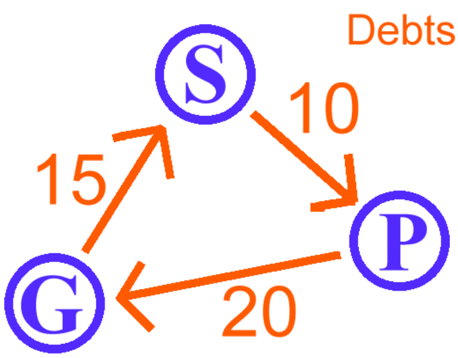
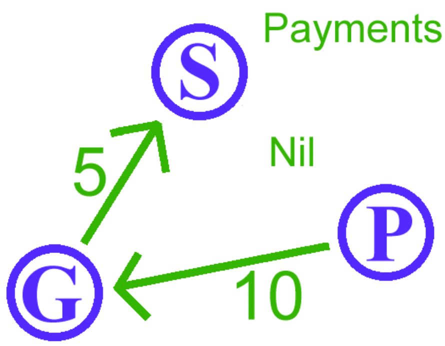
Introduction



If Sally owes Peter £10, Peter owes Geoffrey £20 and Geoffrey owes Sally £15 then one way settle these debts is for each to write out a cheque for the appropriate amount and send it off. This is a blunt method which is capable of improvement. Suppose Peter only has £15 in the bank. He has to wait for Sally to pay him so that he has the funds to write the £20 cheque to Geoffrey. Anyway why should Sally have to write a cheque for £10 when at the end of the day she's expecting to be £5 better off. Suppose that instead of direct settlement Sally asks Geoffrey to pay Peter £10 on her behalf and £5 to her. This solves Sally's irritating problem of having to pay out when she should just be receiving. Now Peter owes Geoffrey £20 and Geoffrey owes Peter £10 resulting in a net debt to Geoffrey of £10 which Peter can pay immediately



Thus we have made three improvements:
(1) The number of transactions is reduced from 3 to 2
(2) The amount of cash committed to the transactions is reduced from £45 to £15.
(3) There was no chain of hold-up due to shortage of cash-flow.
All these results are beneficial to the participants. If the cost of transactions is based on a per-transaction or a commission basis fees are reduced. Settlement can be executed without waiting for a ripple of funds or having to take out temporary loans.

Method

This result has been obtained by transferring debts. We'll now look at the general case and show a method for
(a) reducing the number of transactions to at most the number of participants.
(b) ensuring the reduction of the value of the settlement transactions

Take any two participants (A and B - 'buddies') out of a pool of mutual debtors. Each can say what the net amount due from (owed to) all other participants is. From these two figures we can obtain:
- An inter-buddy settlement between A and B
- A remaining amount due/owed from the other participants to B.

This is done as follows. Suppose we decide that A and B will settle their debt and B will handle all the combined debt to the other participants. It follows that, because A will only be performing a single transaction, the inter-buddy settlement must be for the amount A owes the pool. We also know by how much B is due to be better or worse off after all debts have been settled in the same way (ie sum of all debts). Now we have the additional information that there will be a transaction of a known amount from A. So we can compute the balancing transaction that B will need to execute with the rest of the participants to achieve this gain.

We can pair off all the participants into buddy-pairs and carry out the same procedure.

> A : Debts totalling 78.
> B : Is owed 22.
>
> Since A's debt is being settled via B alone A will 'write a cheque' for 78 to B.
>
> B is now 78 better off when it should be only 22 better off. So it will have to pass on (78-22=) 56 to the remaining participants.
>
> *Note that a preferable arrangement would be for A to be the buddy who deals with the remainder of the participants. Why should B have to 'write a cheque' for 56 when it is owed money (even though it is due to get the money back from A.) If A was to settle with everyone else on behalf of both of them then A writes B a cheque for 22 and the rest of the world will get a cheque from A for 56.*

Now we can pair up these buddy-pairs (AB) with (CD) and so on. If (AB) is going to settle via (CD) then it will have to 'write a cheque' for the full amount that we determined previously. As C settled with D this will have to be directed at D. Since we know what D 'needs' to settle and what B is providing we can work out the difference and 'ask the remaining participants to settle this difference'. We have now reduced four participants to a single entity as far as the rest of the participants are concerned. In essence if B is dealing on behalf of A and D on behalf of C then BD (with the adjusted balances) becomes a buddy-pair. We have in effect dealt with A and C so they can now be ignored.

Note that at no stage have we had to pass information between participants about exactly who owes what to who in any detail. Once we have determined the balancing amount required we can publish this and use it in the next stage of paring.

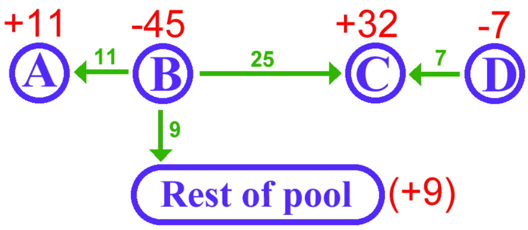
If we had 16 participants there would be 8 pairs, 4 quads, and 2 octets. Each buddy-pair has a single internal transaction, each pair of pairs will have a transaction and so on. As the number of participants increases the total number of transactions approximates to the number of participants.

The worst case number of direct transactions without the benefit of debt swapping is the number of participants squared. (Or half that if A's debt to B and B's debt to A is reduced to a single transaction.) While it is unlikely that every participant would have debts with every other the number of direct transactions would be the number of participants times the average number of business partners. So the debt swapping method reduces the number of transactions by a factor equal to the average number of business partners.

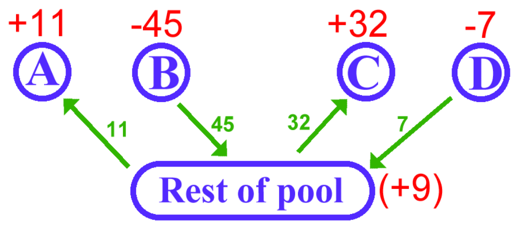
The other linear method of transaction is to have all net debtors pay into a central pool then for all net creditors take from the pool. The number of transactions is equal to the number of participants *but the volume of transactions is not reduced.*

Equalising to reduce volume

The second objective is to reduce the total volume of transactions as well as their number. The ideal situation would be to find pairs (or pairs of pairs etc) which are exactly balanced so that if say A is owed £100 and B owes £100 then these two can settle directly they need not take any further part in the balancing process. We can seek out situations close to this ideal by pairing up the biggest creditor with the biggest debtor then the next biggest creditor and debtor and so on. At each stage the remainders should be getting smaller. The volume of transactions is about half that required for an 'all debtors put in the pool scheme then all creditors take out'.



Left : Example of debt swapping scheme. Amounts owed by the others shown in red. Cash transfers shown in green. Below : Same example using 'debtors put in, creditors take out' scheme.



Notice how the total volume of the transactions in the debt swapping method is 52 compared to 95 in the put in-take out method.

The equalising method just described can easily lead to small players being asked to pass on large amounts which would involve 'writing cheques' when they should just be awaiting receipt. This is not a satisfactory situation but is easily solved. The solution lies firstly in careful selection within a buddy pair of which should be the silent partner. It may be that even this is not sufficient in which case simple graph analysis will indicate where a payment is being made through a small player and a special bypass transaction arranged to cut out this imposition.

To implement an equalising scheme as just described, all debtors have to report their net debt to a server that can sort and pair, work out the balancing transactions and report back to tell those that have to pay how much and who to. This process doesn't require anything more than one 'e-mail' in each direction per participant, and the calculations are extremely simple.

A non-equalising scheme could be implemented without any 'central intelligence' by having a pre-arranged buddy pairing system. Every participant knows in advance who it will be dealing with and can work out it's payments when it has notification from its buddies along an easily determined path. In certain situations a severely unbalanced situation could arise resulting in large transactions being propagated across the network.

Error detection and correction

Error detection in a centrally supervised system is relatively simple. Firstly in any closed system the total debt will always be zero because the creditors will exactly match the debtors.

If some participants have not reported to the centre then these can be treated as a buddy-group of their own and the surplus or deficit from the participants who have reported used as an aggregate figure for their liabilities.

If a participant has mis-reported their total figure then this will be detected as an overall surplus or deficit. Multiple incorrect reporting would normally be caught unless by mis-chance the errors exactly cancelled.

Knowing there is a problem is a very different thing from being able to identify where the problem lies. For a system which operates on the basis of 'all settle together' this could hold up proceedings and make the scheme unworkable. In this case additional information is required. One brute-force approach is to request every participant to report their bottom-line debts with all other participants. These would then be compared and the discrepancies discovered. However where there are a large number of participants this could become rather cumbersome so a cut-down method is proposed: For the sake of reconciliation, participants are divided into two groups and asked to provide their net debt within that group. If there are no errors then the total debt will be zero and the error must lie with the other half of the participants. If the error is in this group then repeat the division process. (Errors could occur in both groups - these would still be detected.) This method, known as "binary chop", allows the source of the error to be identified. However if we were to try and request all this information only on the discovery of a mis-match we could spend ages communicating and waiting for results. A solution would pre-allocate these groups so that participants could provide the check-totals at the time of first submission. These groups do not need to be based on any other grouping. The number of check totals increases by 1 each time the number of participants doubles.

It is an interesting matter how discrepancies would be resolved, as until they are money is being created out of thin air or accumulating in limbo.

Conclusion

A very small amount of information is required to be communicated and that information will always be between fixed points. The schemes described do not carry out the transactions but simply optimises them.

Summary of information to be exchanged between participants and central organiser:

P → C : (1) Balancing amount required
(2) Check totals

C → P : (1) Report of discrepancy and request to negotiate
(2) Make (or expect) payment to X of Y

Simulations of a balancing system show that the number of transactions is typically 5% and volume typically 12% of the best direct one-to-one settlement system. Small numbers of participants reduce these savings but 50% volume and quantity figures are easily obtained with 10 participants.

Finally it should be noted that implementing an efficient debt swapping scheme like this will not be in the interest of existing organisations who provide transaction and financing services. They will suffer loss from a reduction in transactions and a reduction in commission and a reduction in the amount of loans required to service other people's debt.

[Reformatted and contact details updated 4 May 2013]